

# Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

## Vibration Suppression of Flexible Spacecraft During Attitude Maneuvers

Qinglei Hu\* and Guangfu Ma†  
Harbin Institute of Technology,  
150001 Harbin, People's Republic of China

### I. Introduction

VIBRATION reduction is a critical problem related to the maneuvering of spacecrafts, which often employ flexible structures such as solar arrays. Research in this area has been taken along two directions: one direction concentrates on modifying the input command to reduce vibrations, and the other focuses on active suppression of the induced vibrations. Input shaping<sup>1,2</sup> and the component synthesis vibration suppression (CSVS) method<sup>3–5</sup> are two commonly used methods to modify the input command to reduce vibrations of flexible structures. These two methods share the similar principle to reduce the vibration of the flexible structures, whereas the major difference between input shaping and CSVS lies in the design methodology of input command. Input shaping employs complicatedly numerical optimization to derive an input shaper and then convolve it with the reference input commands. In contrast, by the CSVS method the commands are directly designed without solving nonlinear equations, and the CSVS commands can be of many forms.

One promising method for actively suppressing the induced vibrations is to use piezoelectric materials as actuators/sensors because piezoceramics possess the property of piezoelectricity, which describes the phenomenon of generating an electric charge in a material when subjected to a mechanical stress (direct effect) and conversely generating mechanical strain in response to an applied electric field. A wide range of approaches has been proposed for using piezoelectric material to actively control vibration of flexible structures. Positive position feedback (PPF) control was used for active damping of a flexible structure.<sup>6–8</sup> In this approach, the structural position coordinate is fed to the compensator, and the compensator position coordinate, multiplied by a scalar gain, is fed to the structure. Other motivations and benefits in the use of active control technology can also be found in Ref. 9.

In this Note, a new approach integrating CSVS-based command shaping technique and PPF control is proposed for vibration reduction of flexible spacecrafts actuated by on-off thrusters during attitude maneuver. The CSVS method is used to modify the exist-

ing command so that less vibration will be caused by the command itself. In addition, to extend the CSVS method to the system with on-off thrusters, a scheme of combining CSVS method with the pulse-width pulse-frequency (PVPF) modulation<sup>6,8</sup> is first given, which do not require the complicated nonlinear optimizations to design the constant amplitude commands for the system and can suppress the relatively large-amplitude vibrations excited by rapid maneuvers. With the presence of this command shaping control, an additional independent flexible control system, the PPF control technique using piezoelectric materials, acting on the flexible parts can be designed for further vibration suppression during and after the maneuver. Numerical simulations performed on a five-mode model of the spacecraft with flexible appendages during rest-to-rest maneuver demonstrate the theoretical and practical merit of the proposed approach.

### II. Dynamic Modeling of a Slewing Active Structure

Figure 1 shows the model of a flexible spacecraft, which consists of a rigid hub with radius  $b$ , a uniform cantilever flexible beam with surface-bonded piezoelectric sensors and actuators, the length  $l$ , and the tip mass  $m_t$ . Define the OXY and oxy as the inertial frame and the frame fixed on hub, respectively. The attitude angle  $\theta$  denotes the relative motion of these frames. Denotes  $W(x, t)$  as the flexible deformation at  $x$  with respect to the oxy frame, where  $x \in [0, l]$ .

For system modeling, several assumptions are made: 1) the beam is assumed to be an Euler–Bernoulli beam and flexible only in a direction transverse to its length in the plane of motion; 2) the piezoelectric layer is homogeneous and is uniaxially polarized; 3) the piezoelectric material is perfectly bonded to the beam; and 4) the gravitational effect, hub dynamics, and internal/external disturbances are neglected for simplicity.

Now, applying the extended Hamilton's principle, the equations of motion for the flexible spacecraft with surface-bonded piezoceramics are<sup>7,8</sup>

$$J\ddot{\theta} + \sum_{i=1}^n D_i \dot{q}_i = T \quad (1)$$

$$\ddot{q} + D\ddot{\theta} + Z\dot{q} + \lambda q = -Be_a \quad (2)$$

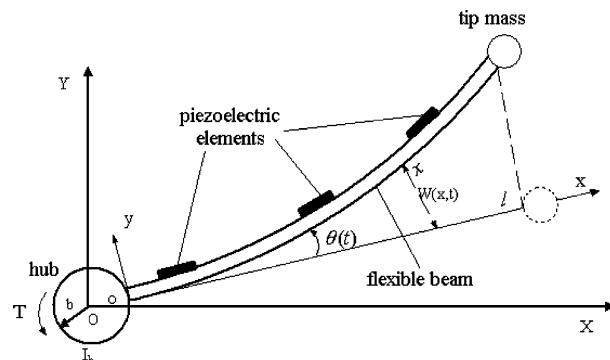


Fig. 1 Model of flexible spacecraft with surface-bonded piezoelectric material.

Received 22 August 2004; revision received 12 November 2004; accepted for publication 17 November 2004. Copyright © 2004 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved. Copies of this paper may be made for personal or internal use, on condition that the copier pay the \$10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 0731-5090/05 \$10.00 in correspondence with the CCC.

\*Research Assistant, Department of Control Science and Engineering, 92 Xidazhi Street.

†Distinguished Professor of Aerospace Engineering, Department of Control Science and Engineering, 92 Xidazhi Street.

$$\gamma e = B^T q \quad (3)$$

where  $J$  is the flexible spacecraft moment of inertia,  $q_i$  is the modal coordinate,  $q$  is the modal coordinate vector  $[q_1 \ q_2 \ \dots \ q_n]^T$ ,  $D_i$  is the rigid-elastic coupling for each vibration mode,  $D$  is the rigid-elastic coupling vector  $[D_1 \ D_2 \ \dots \ D_n]^T$ ,  $Z$  is the modal damping matrix  $\text{diag}\{2\xi_i\omega_i\}$ , and  $\lambda$  is the matrix of natural frequencies  $\text{diag}\{\omega_i^2\}$ .  $B^T$  is the electromechanical coupling term,  $\gamma$  is the constant of the piezoelectric material, and  $e$  is the electrical voltage.

### III. Control Strategy

Figure 2 presents a block diagram of the proposed control scheme. The control system for vibration reduction of the flexible spacecraft during attitude maneuvers consists of two subsystems, the attitude control subsystem using PWWF modulation and the active vibration suppression subsystem using piezoelectric materials. The PWWF modulation is used to control thruster firing, and the attitude feedback control subsystem also employs a proportional plus derivative (PD) controller, which is used to follow the shaped command. The active vibration suppression subsystem uses PPF control strategy, which implements the lead zirconate titanate (PZT) sensors and actuators to actively cancel the thruster-firing-induced vibration. These two subsystems work together to reduce vibrations during attitude maneuver.

#### A. CSVS Method

In this section, the principle of the CSVS method<sup>3–5</sup> is discussed briefly, which is a vibration self-canceled method by which the vibration excited by the former input can be canceled by the other inputs with suitable time delays. Considering the following Eq. (4), which is a generalized vibration system:

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2x = F(t) \quad (4)$$

where  $x$  is the state coordinate,  $\omega$  is the natural frequency,  $\zeta$  is the damping ratio, and  $F(t)$  is the control input. The damped vibration frequency can be represented by  $\omega_d = \omega\sqrt{1 - \zeta^2}$ , and the corresponding period is  $T_d = 2\pi/\omega_d$ . Figure 3 demonstrates the simplest example of application of CSVS method. The vibration excited by impulse 1 at time 0, with the amplitude  $A$ , is cancelled by impulse 2 implemented at time  $T_d/2$  with the amplitude  $Ae^{-\pi\zeta\sqrt{1-\zeta^2}}$ . Ideally, no vibration exists after such superimposition.

The following Lemma 1 gives the principle of CSVS method to design the component sequences:

**Lemma 1:** Give a vibration mode with the natural frequency  $\omega$ , period  $T_d$ , and damping ratio  $\zeta$ . Implementation of  $n$  (the number of the input components  $n$  can be any positive integer) similar components (the components can be in the form of either impulses or time variable functions), whose amplitudes are scaled by an attenuation factor of  $e^{-\zeta\omega t}$  at  $n$  time instants of  $0, T_d/n, \dots, (n-1)T_d/n$ , leads to suppression of this vibration mode completely.

Ideally, all vibrations can be canceled after applying the CSVS commands, provided that these two parameters can be known exactly. In practice, however, as a result of estimation errors of these

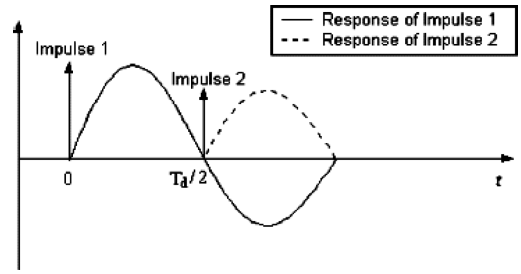


Fig. 3 Basic principle of CSVS method.

two parameters, vibration can still exit after applying the CSVS commands. The robust CSVS command<sup>5</sup> can be recognized by the derivative of the system response with respect to the parameter of vibration frequencies or damping ratios.

**Lemma 2:** If a CSVS input command can suppress a vibration mode with the  $p$ th-order robustness to the frequency estimation error, then the new command, formed by synthesizing  $n$  of such commands according to Lemma 1, can suppress the same vibration mode, but with the  $p+1$ th-order robustness to the frequency error.

The robustness to uncertainty of the damping ratio can be analyzed in a similar manner because the derivative of the system response with respect to the frequency has the same expression as that with respect to damping ratio.

The CSVS method can also work on suppression of multiple modes of vibrations. The principle of constructing the multimode CSVS commands is similar to that of constructing the robust CSVS commands. The following Lemma 3 gives the principle of constructing the multimode CSVS commands.

**Lemma 3:** If a CSVS input command can suppress all of the  $m-1$  vibration modes, then the new command, formed by synthesizing  $n$  of such commands according to lemma 1 to suppress the  $m$ th vibration mode, can suppress all the  $m$  vibration modes.

According to Lemmas 1–3, various CSVS commands can be constructed, with any number of components  $n$ , any order of robustness, and any multiple modes. However, the number of components  $n$  plays an important role in CSVS commands. If  $n$  is too large, considerable online computational work will be needed, although the higher-order robustness can be enhanced. If  $n$  is too small, the robustness for higher-order modes will be degraded.

#### B. Positive Position Feedback Control

For the control of flexible structures, the PPF control scheme is well suited for implementation utilizing the piezoelectric sensors and actuators. This approach has several desirable characteristics: it is insensitive to spillover, it offers quick damping for a particular mode, and it is easy to implement.

The equations describing PPF control operation are given as<sup>7</sup>

$$\ddot{\eta}_s + D_s \dot{\eta}_s + \Omega_s \eta_s = a_1 C^T G \xi \quad (5)$$

$$\ddot{\xi} + D_f \dot{\xi} + \Omega_f \xi = a_2 \Omega_f C \eta_s \quad (6)$$

where  $\eta_s$  is the modal coordinate vector describing displacement of the structure,  $D_s$  is the modal damping matrix of the structure,  $\Omega_s$  is the modal frequency matrix of the structure,  $a_1$  is a constant related to actuator sensitivity,  $G$  is the feedback gain matrix,  $\xi$  is the compensator coordinate vector,  $D_f$  is the compensator damping matrix,  $\Omega_f$  is the compensator frequency matrix,  $a_2$  is constant representing sensor sensitivity, and  $C$  is a fully populated participation matrix, which determines the influence of each sensor/actuator pair on each compensator and vice versa.

The selection of PPF gains is dictated by a stability criterion, which is in the form of positive definiteness of a matrix consisting of feedback gains and system parameters as follows.

**Lemma 4:** The stability condition for the two combined systems (5) and (6) can be written as<sup>7</sup>

$$\Omega_s - a_1 a_2 C^T G C > 0 \quad (7)$$

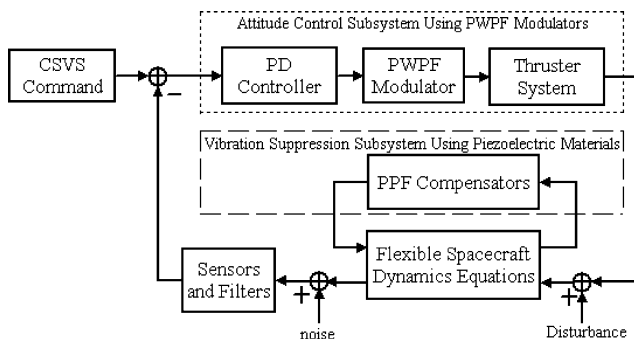


Fig. 2 Block diagram of control system for vibration reduction during attitude maneuver.

The feedback gain matrix  $G$  consists of each feedback gain, which is associated with each flexible mode.

#### IV. Simulation Results and Comparisons

The spacecraft simulation involves a single-axis rest-to-rest maneuver. The parameters for the simulated flexible spacecraft are given in the Ref. 5. Because the goal is to suppress vibrations of low-frequency modes, the first two modes of five, 3.161 rad/s, and 16.954 rad/s with all modal ratios of 0.004, respectively, are major

concerns for vibration suppression. The type of PZT-5A piezoelectric ceramic plates<sup>6</sup> is bonded to the surface of the flexible appendage. The PPF compensator parameters are selected as  $g_1 = 0.5$ ,  $g_2 = 0.5$ ,  $\omega_{c1} = 2.5$ ,  $\omega_{c2} = 16$ ,  $\zeta_{c1} = 0.5$ , and  $\zeta_{c2} = 0.5$ . The proportional gain and the derivative gain of the PD controller for attitude control are 15 and 50, respectively. Eight components, second-order robust with four components for the first mode and zero-order robust with two components for the second, are designed according to the CSVS principle as shown in Sec. III.A.

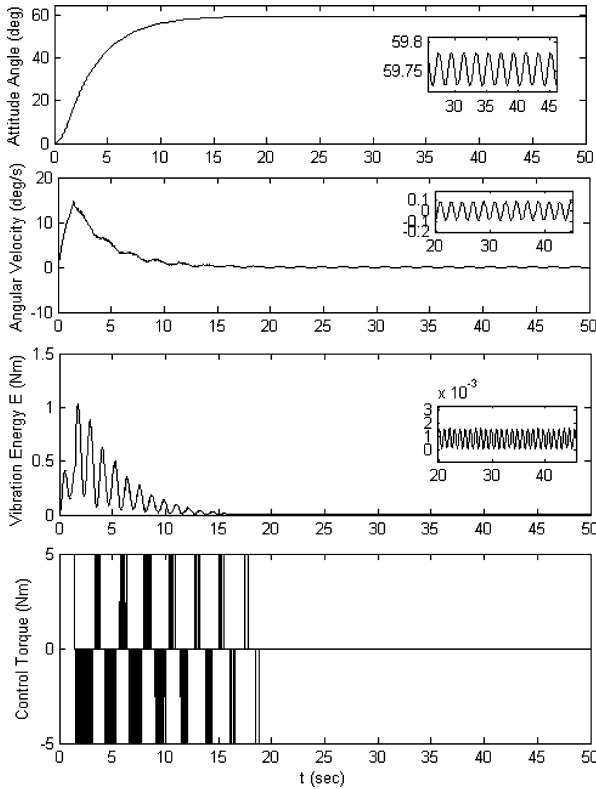


Fig. 4 Slewing with PD control without CSVS method.

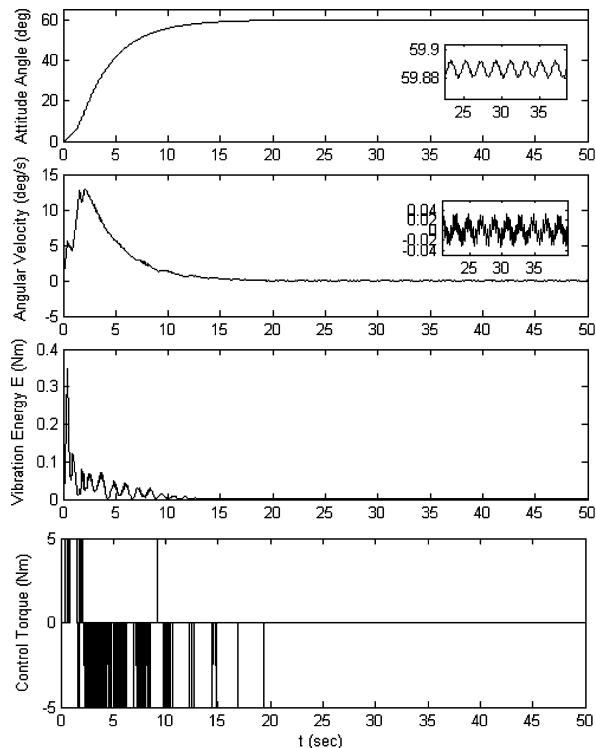


Fig. 5 Slewing with PD control with CSVS method.

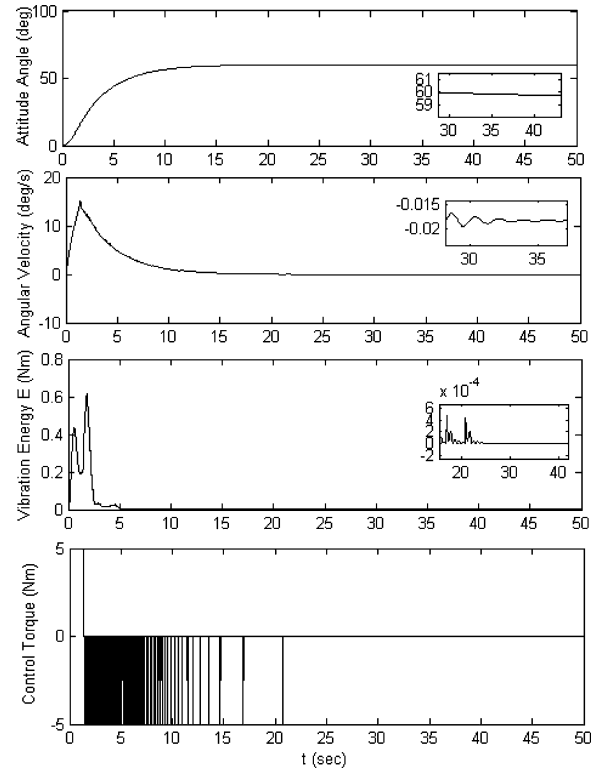


Fig. 6 Slewing with PD control with PPF control.

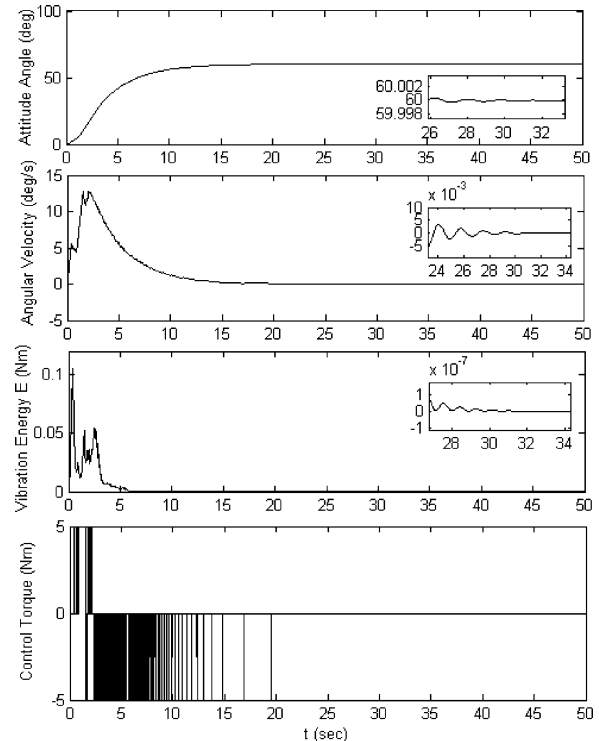


Fig. 7 Slewing with PD control with a second-order robust for the first mode, zero-order robust for the second mode and PPF control.

In this simulation, for comparative purposes, four cases of a 60-deg slew of the flexible are conducted: 1) slew using PD without CSVS method, 2) slew using PD with CSVS method, 3) slew using with PD control with PPF control, and 4) slew using PD control with a second-order robust for the first mode, zero-order robust for the second mode and PPF control. Case 1 is compared with case 2 to show the effectiveness of CSVS method. Plots of angle and angular velocity, vibration energy level  $E = \dot{q}^T \dot{q} + q^T \lambda q$ , and the applied control torques are shown in Figs. 4 and 5, respectively. To demonstrate the effect of the active vibration suppression using PZT sensors and actuators, the active vibration suppression subsystem is now turned on. Time histories of cases 3 and 4 are shown in Figs. 6 and 7, respectively. Because the PPF control only targets as the first and second modes, it is reasonable to see that the higher modes vibrate at the same energy level in either case.

## V. Conclusions

In this Note, a new approach for vibration reduction of flexible spacecraft during attitude maneuver is presented. The new approach integrates the method of component synthesis vibration suppression (CSVs) and the technique of positive position feedback (PPF) control with the PZT sensors and actuators. By using the pulse-width pulse-frequency (PWPF) modulation for thruster control, the CSVS method can be extended to the system with on-off actuators and can suppress the relatively large amplitude vibrations excited by rapid maneuvers. The technique of PPF control with PZT sensors and actuators is used to actively suppress the microvibrations during and after the slew. Simulation results demonstrate that the new approach can significantly reduce the vibration of the flexible appendages during maneuver operations.

## Acknowledgments

The authors thank Gangbing Song and Dun Liu for their useful design and insightful comments. Moreover, we thank the anonymous reviewers for their helpful comments.

## References

- <sup>1</sup>Singer, N., and Seering, W., "Presaping Command Inputs to Reduce System Vibration," *Journal of Dynamic Systems, Measurement and Control*, Vol. 112, No. 3, 1990, pp. 76–82.
- <sup>2</sup>Singhose, W., Pao, L., and Seering, W., "Time-Optimal Rest-to-Rest Slewing of Multi-Modes Flexible Spacecraft Using ZVD Robustness Constraints," AIAA Paper 96-3845, July 1996.
- <sup>3</sup>Liu, D., Yang, D. M., Xi, J., and Zhang, W. P., "An Optimal Maneuver Control Method for the Spacecraft with Flexible Appendage," AIAA Paper 88-4255, Aug. 1988.
- <sup>4</sup>Liu, D., and Wu, G. Y., "An Active Vibration Suppression Method for the Flexible Spacecraft and Its Application," AIAA Paper 94-3757, Aug. 1994.
- <sup>5</sup>Shan, J. J., Sun, D., and Liu, D., "Design for Robust Component Synthesis Vibration Suppression of Flexible Structures with On-Off Actuators," *IEEE Transactions Robotics and Automation*, Vol. 20, No. 3, 2004, pp. 512–525.
- <sup>6</sup>Song, G., and Agrawal, B. N., "Vibration Suppression of the Flexible Spacecraft During Attitude Control," *Acta Astronautica*, Vol. 49, No. 2, 2001, pp. 73–83.
- <sup>7</sup>Fanson, J. L., and Caughey, T. K., "Positive Position Feedback Control for Large Structure," *AIAA Journal*, Vol. 28, No. 4, 1990, pp. 717–724.
- <sup>8</sup>Song, G., Buck, N., and Agrawal, B., "Spacecraft Vibration Reduction Using Pulse-Width Pulse-Frequency Modulated Input Shaper," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 3, 1999, pp. 433–440.
- <sup>9</sup>Hyland, D. C., Junkins, J. L., and Longman, R. W., "Active Control Technology for Large Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 5, 1993, pp. 801–821.